You know, for a mathematician, he did not have enough imagination. But he has become a poet and now he is fine.

Name and section: ______________________________

1. Let $T : \mathbb{P}_2 \to \mathbb{P}_2$ be the linear transformation $T(p(t)) = p(t+1) - p(t)$.
   (a) (1 point) $B = \{1, t, t^2\}$ is a basis of $\mathbb{P}_2$. Find the matrix ${}_B[T]_B$.
   (b) (2 points) $C = \{1, t, \frac{t(t-1)}{2}\}$ is another basis of $\mathbb{P}_2$. Find the matrix ${}_C[T]_C$.
   (c) (2 points) Find the eigenvalues of T. Is T diagonalizable?

2. Below all matrices are $n \times n$ with real entries unless otherwise specified. Label the following statements as true or false. (You do not need to justify your answers.)

   (a) (1 point) If A and B have the same characteristic polynomial, then they are similar.

   (b) (1 point) If A and B are similar, then they have the same eigenvectors.

   (c) (1 point) If $A = P^{-1}DP$ where D is diagonal, then the columns of P are eigenvectors of A.

   (d) (1 point) If a 2×2 real matrix A has complex eigenvalues $a \pm bi$, then $\det(A) = a^2 + b^2$.

   (e) (1 point) It is possible for a 4×4 matrix to have no real eigenvalues.